Name $\qquad$
CS 383
Final Exam
May 19, 2007

1. (15 points) Short Answers:
a) Suppose $\mathcal{L}_{1}$ and $\mathcal{L}_{2}$ are both Recursively Enumerable. Is their intersection Recursively Enumerable? Why?
b) What is the Post Correspondence Problem? Why do we care about it?
c) In 1900 the mathematician David Hilbert asked if there was any problem that couldn't be solved algorithmically. How would you answer that question? Give some justification for your answer (it doesn't need to be a full proof, but at least relate your answer to what we did in class).
2. (20 points) Classify the following languages as Regular, Context Free, Decidable, Recursively Enumerable, or non-RE. It is sufficient to use the most specific category for each language: if you say one is Regular, you don't need to also say it is Context Free and RE. You don't need to justify your answers.
a) The set of names of all people who have ever taught a course in Theory of Computer Science anywhere in the world.
b) $\left\{0^{\mathrm{n}} 10^{\mathrm{m}} \mid \mathrm{n}>=0 ; \mathrm{n}<=\mathrm{m}<=2 \mathrm{n}\right\}$
c) $\left\{0^{n} 0^{m} 1 \mid \mathrm{n}>=0 ; \mathrm{n}<=\mathrm{m}<=2 \mathrm{n}\right\}$
d) $\left\{0^{\mathrm{n}} 1^{\mathrm{n}} 2^{\mathrm{m}} 3^{\mathrm{m}} \mid \mathrm{n}>=0\right.$ and $\left.\mathrm{m}>=0\right\}$
e) The set of descriptions of DFA's that accept the string " 010 ".
f) The set of descriptions of TM's that accept the string " 010 ".
g ) The set of pairs ( $\mathrm{M}, \mathrm{w}$ ) where M is a Turing Machine and w is an input string that makes M loop forever.
3. (10 points) Convert the following $\varepsilon$-NFA to a DFA. What language does it accept?

4. (10 points) Give a PDA that accepts strings of 0 's and 1 's with more 0 's than 1 's. Note that 000,001 , and 1110000 are all strings in this language but 01 is not.
5. (20 points) Show that $\left\{w^{\mathrm{w}} \mathrm{w}^{\mathrm{R}} \mid \mathrm{w}\right.$ is a string of 0 's and 1 's $\}$ is Context Free, but $\{\mathrm{w} w \mid \mathrm{w}$ is a string of 0 's and 1 's $\}$ is not. Note that 0110 is a string in the first language and 0101 is a string in the second.
6. (10 points) The SubsetSum Problem is: Given a set $\left\{\mathrm{x}_{1} \ldots \mathrm{x}_{\mathrm{n}}\right\}$ of positive numbers and a number M , is there a subset of the numbers that sums to M ? The Partition Problem is: Given a set $\left\{\mathrm{x}_{1} . . \mathrm{x}_{\mathrm{n}}\right\}$ of positive numbers, can we partition this set into two sets whose sums are the same? Use the fact that Partition is NP-Complete to show that SubsetSum is also NP-Complete.
7. (15 points) The Halting Language is the set $\{(M, w) \mid M$ is a Turing Machine, $w$ is an input and M halts on w$\}$.
a) Use the fact that the universal language $\mathcal{L}_{\mathrm{u}}$ is not decidable to prove that the Halting Language is also not decidable.
b) Is the Halting Language Recursively Enumerable? Why?

Please write and sign the Honor Pledge.

